Imprints of the nuclear symmetry energy on gravitational waves from the axial w-modes of neutron stars

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The eigen-frequencies of the axial w-modes of oscillating neutron stars are studied using the continued fraction method with an Equation of State (EOS) partially constrained by the recent terrestrial nuclear laboratory data. It is shown that the density dependence of the nuclear symmetry energy $E_{sym}(\rho)$ affects significantly both the frequencies and the damping times of these modes. Besides confirming the previously found universal behavior of the mass-scaled eigen-frequencies as functions of the compactness of neutron stars, we explored several alternative universal scaling functions. Moreover, the w_{II}-mode is found to exist only for neutron stars having a compactness of $M/R \geq 0.1078$ independent of the EOS used.

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I. INTRODUCTION

Presently, among the greatest challenges in modern physics are determining the equation of state (EOS) of dense neutron-rich matter and detecting gravitational waves - tiny ripples in space-time predicted by the theory of general relativity. Physics of neutron stars provides a unified framework for studying both phenomena: on the one hand neutron stars are natural sites of super-dense matter, while on the other hand they are one of the major candidates for emitting gravitational waves in the bandwidth targeted by the ground based laser interferometric detectors LIGO [1], VIRGO (e.g. [2]) and GEO [3]. Besides the gravitational waves from elliptically deformed pulsars, an important mechanism of gravitational radiation from a neutron star is its non-radial oscillations. The latter may provide potentially a unique probe to the EOS of super-dense neutron-rich matter because the resulting gravitational waves carry important information about the neutron star structure [4, 5, 6]. The non-radial neutron star oscillations could be triggered by various mechanisms such as gravitational collapse, a pulsar "glitch" - a sudden disrupt of the otherwise very regular pulsar period, gravitational spin-orbit coupling due to a companion in a close binary orbit, or a transition to novel phases of matter in the inner core.

In the framework of general relativity, gravitational radiation damps out the neutron star oscillations. The frequency of the non-radial oscillations thus becomes "quasi-normal" (complex) with a real part representing the actual frequency of the oscillation and an imaginary part representing the energy losses. The eigen-frequencies of the quasi-normal modes can be found by solving the equations describing the non-radial perturbations of a static neutron star in general relativity [7, 8]. These equations are derived by expanding the perturbed tensors in spherical harmonics. The perturbation equations split into two sets: one for the axial mode for which the spherical harmonics transform under parity (i.e., under the transformation $\theta \to \pi - \theta$ and $\phi \to \pi + \phi$) as $(-1)^{l+1}$; and second for the polar mode for which the spherical harmonics transform under parity as $(-1)^l$, respectively. With the appropriate boundary conditions at the neutron star center and surface, the solution of the perturbation equations yields the complex eigen-frequencies. They can be classified as follows: (1) f-modes, associated with the global oscillation of the fluid; (2) g-modes, associated with the fluid buoyancy; and (3) p-modes, associated with the pressure gradient. These modes exist in both Newtonian gravitational theory and general relativity. There is still another type of mode that only exists in general relativity - the so called w-mode associated with space-time and for which the motion of the fluid is negligible [4, 7, 8]. The w-mode is very important for astrophysical applications since it is related to the space-time curvature and exists for all relativistic stars, including black holes. The standard axial w-mode is categorized as w_I. Additionally, there exists an interesting family of axial w-modes [9], categorized as w_{II}. According to the work of Chandrasekhar & Ferrari [7, 8], the axial w-mode is described by a unique second-order differential equation. Its simple form makes it possible to conduct detailed numerical studies of its spectrum. A major characteristic of the axial w-mode is its high frequency accompanied by very rapid damping. In fact, the w-mode frequencies are outside the peak of the detector sensitivities of the currently operating and planned gravitational wave detectors. Nevertheless, it is theoretically interesting to investigate the w-mode of neutron star oscillations as it is an important mechanism for producing gravitational waves [7, 8, 9, 10, 11]. Hopefully, the

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study will help stimulate discussions on detecting high-frequency gravitational waves with new generations of detectors in the future.

The gravitational wave frequency of the axial w-mode depends on the structure and properties of neutron stars [9], which are determined by the EOS of neutron-rich stellar matter. However, at present time the EOS of matter under extreme conditions (densities, pressures and isospin asymmetries) is still rather uncertain and theoretically controversial. One of the main sources of uncertainties in the EOS of neutron-rich matter is the poorly known density dependence of the nuclear symmetry energy, $E_{sym}(\rho)$ [12]. The EOS of neutron-rich nuclear matter plays an important role not only in many astrophysical objects/processes [13] but also in heavy-ion collisions especially those induced by neutron-rich radioactive beams in terrestrial laboratories [14]. While heavy-ion collisions are not expected to create the same matter and conditions as in neutron stars, the same elementary nuclear interactions are at work in the two cases [15]. Thus, it is important to examine ramifications of conclusions regrading the EOS extracted from one field in the other one. On one hand, extremely impressive progress has been made in astrophysical observations relevant for constraining the EOS of nuclear matter. To our best knowledge, however, mainly because of the low precision associated with the current measurements of neutron star radii, a non-controversial conclusion on the EOS and the density dependence of the nuclear symmetry energy has yet to come from analyzing astrophysical observations. Nevertheless, it is very interesting to note that since the pioneering work of Lindblom [16], a lot of efforts have been devoted to extracting the underlying EOS by using the technique of inverting the TOV (Tolman-Oppenheimer-Volkov) equation using the masses (M) and radii (R) simultaneously measured accurately for several neutron stars, see, e.g., refs. [17, 18], for the latest reports. While the principle of this technique is well demonstrated and promising, the lack of simultaneously measured (M, R) data has so far hindered the fruitful applications of this technique. In particular, while the masses in some binary systems, such as the double neutron star binaries, are well measured, their radii are unfortunately not precisely known [17, 18]. Similarly, under the premise that the frequencies and damping rates of several w-modes of a single neutron star are measured, Tsui and Leung [19] investigated an inversion scheme to determine the mass, radius and density profile of the neutron star. The underlying EOS can then also be inferred. While the approach is convincingly promising, it can only be applied after the gravitational wave astronomy becomes a reality. On the other hand, heavy-ion reactions especially those induced by radioactive beams provide an alternative means to constrain the EOS of neutron-rich nuclear matter, see, e.g., refs.[14, 20, 21, 22, 23], for reviews. Over approximately the last 40 years, the nuclear physics community has made steady progress in constraining the EOS of symmetric nuclear matter up to several times the normal nuclear matter density. However, it was only during roughly the last 10 years the nuclear physics community has made significant progress in constraining the density dependence of the symmetry energy of neutron-rich nuclear matter thanks mainly to the rapid technical advantages in accelerating radioactive beams [14]. It is particularly exciting to see that some significant results in constraining the density dependence of the symmetry energy have beeb reported very recently [24, 25, 26, 27]. Thus, it is interesting to investigate how the EOS constrained by the available data from heavy-ion reactions may help limit the ranges of frequencies and damping times of the w-mode and the strain of other forms of gravitational waves [28]. In this work we apply an EOS with its symmetric part constrained up to about 5 times the normal nuclear matter density and its symmetry energy constrained in the subsaturation density region by the available heavy-ion reaction data. We have two major objectives: (1) to investigate to what extent the nuclear symmetry energy could affect the eigen-frequency of axial w-mode; and (2) to examine the relationship between the axial w-mode eigen-frequency and the neutron star gravitational field. This study is important because the axial w-mode is believed to be a pure space-time mode, its eigen-frequency is expected to have a more direct relationship with the properties of gravitational field, e.g. with the energy of gravitation. Compared to previous studies on the w-mode in the literature, the major issues we address here are scientifically important and studied from a different direction. In light of the great importance of and the extreme difficulties encountered in detecting gravitational waves, our results presented here are scientifically useful.

This paper is organized as follows: after the introduction, in Section II we recall the formalism and the continued fraction method for computing the eigen-frequency of the axial w-mode (the set of perturbation equations and their numerical solution); we then briefly outline the EOSs applied in our studies in Section III; in Section IV we present our numerical results; and in Section V we conclude with a short summary.

II. FORMALISM FOR CALCULATING THE AXIAL W-MODE FREQUENCY

In what follows we summarize the formalism for calculating the eigen-frequency of the axial w-mode and the continued fraction method. Unless noted otherwise, we use geometrical unit ($G = c = 1$). According to Chandrasekhar & Ferrari [7, 8], the axial perturbation equations for a static neutron star can be simplified by introducing a function $z(r)$, constructed from the radial part of the perturbed axial metric components. It satisfies the following differential equation

$$\frac{d^2 z}{dr_*^2} + [\omega^2 - V(r)]z = 0, \tag{1}$$

where $\omega(=\omega_0+i\omega_i)$ is the complex eigen-frequency of the axial w-mode. Inside the star, the tortoise coordinate r_* and the potential function V are defined by

$$r_* = \int_0^r e^{\lambda-\nu} dr \ (or \ \frac{d}{dr_*} = e^{\lambda-\nu}\frac{d}{dr}) \qquad (2)$$

and

$$V = \frac{e^{2\nu}}{r^3}[l(l+1)r + 4\pi r^3(\rho - p) - 6m] \qquad (3)$$

with l the spherical harmonics index (used in describing the perturbed metric, here only the case $l = 2$ is considered), ρ and p the density and pressure, and m the mass inside radius r, respectively. The functions e^ν and e^λ are given by the line element for a static neutron star as

$$-ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (4)$$

Outside the neutron star, Eqs. 2 and 3 reduce to

$$r_* = 2M\ln(r - 2M) \ (or \ \frac{d}{dr_*} = \frac{r-2M}{r}\frac{d}{dr}) \qquad (5)$$

and

$$V = \frac{r-2M}{r^4}[l(l+1)r - 6M]. \qquad (6)$$

where M is the total gravitational mass of neutron star. The solutions to this problem are subject to a set of boundary conditions (BC) constructed by Chandrasekhar & Ferrari [7] - regular BC at the neutron star center, continuous BC at the surface and behaving as a purely outgoing wave at infinity.

As discussed in references [4, 9, 30], the w-modes have larger imaginary parts and this introduces considerable difficulties to dealing with the boundary conditions at infinity. For a larger radius, the term representing the incoming wave, $z^{in} \sim e^{-r_*\omega_i}$, vanishes and therefore the problem no longer satisfies the necessary BC. A practical way to overcome these difficulties is to utilize the *continued fraction method* [9, 30], which is applicable to both the axial and polar w-modes. This method provides both the w_I- and w_{II}-modes.

Here we give a brief introduction to the continued fraction method following closely Benhar et al. [9]. It is convenient to use the dimensionless geometrical units, i.e. $2M = c = G = 1$. First, one represents the solution outside the neutron star as

$$z(r) = (r-1)^{-i\omega}e^{-i\omega r}\sum_{n=0}^{\infty} a_n y^n \equiv \chi(r)\sum_{n=0}^{\infty} a_n y^n, \qquad (7)$$

where $y = 1 - a/r, R < a < 2R$ (R is the stellar radius) and the coefficients a_n satisfy the following four-term recurrence relation

$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} + \delta_n a_{n-2} = 0; \ (n \geq 2) \qquad (8)$$

with

$$\alpha_n = (1 - \frac{1}{a})n(n+1), \qquad (9)$$

$$\beta_n = -2(i\omega a + n - \frac{3n}{2a})n, \qquad (10)$$

$$\gamma_n = (1 - \frac{3}{a})n(n-1) + \frac{3}{a} - l(l+1), \qquad (11)$$

$$\delta_n = \frac{1}{a}(n-3)(n+1). \qquad (12)$$

The coefficients a_0 and a_1 are determined by the continuity of $z(r)$ and $z(r)_{,r}$ at $r = a$, that is

$$a_0 = \frac{z(a)}{\chi(a)}, \qquad (13)$$

$$a_1 = \frac{a}{\chi(a)}[z_{,r}(a) + \frac{i\omega a}{a-1}z(a)]. \qquad (14)$$

Since Eq. 8 is a four-term recurrence relation, to determine a_n uniquely one must know three initial terms, i.e. Eqs. 13 and 14 are not sufficient alone. Leaver [31] has shown that Eq. 8 can be further reduced to a three-term recurrence relation as

$$\hat{\alpha}_n a_{n+1} + \hat{\beta}_n a_n + \hat{\gamma}_n a_{n-1} = 0, \ (n \geq 2), \qquad (15)$$

for $n = 0$

$$\hat{\alpha}_0 = -1, \ \hat{\beta}_0 = \frac{a_1}{a_0}, \qquad (16)$$

for $n = 1$

$$\hat{\alpha}_1 = \alpha_1, \ \hat{\beta}_1 = \beta_1, \ \hat{\gamma}_1 = \gamma_1, \qquad (17)$$

for $n \geq 2$

$$\hat{\alpha}_n = \alpha_n, \ \hat{\beta}_n = \beta_n - \frac{\hat{\alpha}_{n-1}\delta_n}{\hat{\gamma}_{n-1}}, \ \hat{\gamma}_n = \gamma_n - \frac{\hat{\beta}_{n-1}\delta_n}{\hat{\gamma}_{n-1}}. \qquad (18)$$

According to Leaver [31] and Wall [32], the function $z(r)$ describes a purely outgoing wave at infinity only if

$$f_n(\omega) = \hat{\beta}_n - \frac{\hat{\alpha}_{n-1}\hat{\gamma}_n}{\hat{\beta}_{n-1} - \frac{\hat{\alpha}_{n-2}\hat{\gamma}_{n-1}}{\hat{\beta}_{n-2} - \dots \frac{\hat{\alpha}_0\hat{\gamma}_1}{\hat{\beta}_0}}} - \frac{\hat{\alpha}_n\hat{\gamma}_{n+1}}{\hat{\beta}_{n+1} - \frac{\hat{\alpha}_{n+1}\hat{\gamma}_{n+2}}{\hat{\beta}_{n+2} - \frac{\hat{\alpha}_{n+2}\hat{\gamma}_{n+3}}{\hat{\beta}_{n+3} - \dots}}} = 0 \ (n = 0, 1, 2, ...). \quad (19)$$

For $n = 0$, the above expression becomes

$$f_0(\omega) = \hat{\beta}_0 - \frac{\hat{\alpha}_0\hat{\gamma}_1}{\hat{\beta}_1 - \frac{\hat{\alpha}_1\hat{\gamma}_2}{\hat{\beta}_2 - \frac{\hat{\alpha}_2\hat{\gamma}_3}{\hat{\beta}_3 - \dots}}} = 0. \quad (20)$$

Eq. 19 is the key expression for computing the eigen-frequencies of the axial w-modes. Obviously $f_0(\omega) = 0$ is the simplest case, which is mainly employed in our calculation. In order to rule out the pseudo roots, the cases $n = 1$ and 2 are also examined.

The numerical procedure goes as follows: (1) choose a suitable range for ω_0 and ω_i; (2) integrate Eq. 1 starting at the center to $r = a$; (3) compute $f_0(\omega)$ through Eq. 20 and plot the curve ω_0 against ω_i (where the real part and the imaginary part of function $f_0(\omega)$ are zero); (4) determine the crossing point of the curves - the crossing point specifies the eigen-frequency of the axial w-mode. The same procedure is also repeated for the cases with $n = 1$ and $n = 2$.

III. THE EQUATION OF STATE OF NEUTRON-RICH NUCLEAR MATTER PARTIALLY CONSTRAINED BY RECENT TERRESTRIAL NUCLEAR LABORATORY DATA

The necessary input in solving the Eq. 1 is the EOS. While at high densities reached in the core of neutron stars, other particles and new phases of matter may exist, in this work we consider the simplest model of neutron stars consisting of neutrons, protons and electrons (*npe*) in beta-equilibrium. For isospin asymmetric nuclear matter, various theoretical studies have shown that the energy per nucleon can be well approximated by

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + O(\delta^4) \quad (21)$$

in terms of the baryon density $\rho = \rho_n + \rho_p$, the isospin asymmetry $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, the energy per nucleon in symmetric nuclear matter $E(\rho, \delta = 0)$, and the bulk nuclear symmetry energy $E_{sym}(\rho)$ [12, 33, 34, 35]. The corresponding pressure of the *npe* matter is given by

$$\begin{aligned} P(\rho, \delta) &= \rho^2 \left(\frac{\partial E}{\partial \rho}\right)_\delta + \frac{1}{4}\rho_e\mu_e \\ &= \rho^2 \left[E'(\rho, \delta = 0) + E'_{\rm sym}(\rho)\delta^2\right] \\ &+ \frac{1}{2}\delta(1-\delta)\rho E_{\rm sym}(\rho), \end{aligned} \quad (22)$$

where $\rho_e = \frac{1}{2}(1-\delta)\rho$ and $\mu_e = \mu_n - \mu_p = 4\delta E_{\rm sym}(\rho)$ are, respectively, the density and chemical potential of electrons. It is obvious that the pressure is dominated by the symmetry energy term in the *npe* matter near the saturation density where the $E'(\rho, \delta = 0)$ term vanishes. The value of the isospin asymmetry δ at β equilibrium is determined by the chemical equilibrium and charge neutrality conditions, i.e., $\delta = 1 - 2x_p$ with

$$x_p \approx 0.048 \left[E_{\rm sym}(\rho)/E_{\rm sym}(\rho_0)\right]^3 (\rho/\rho_0)(1 - 2x_p)^3. \quad (23)$$

It is seen that the proton fraction x_p is uniquely determined by the density dependence of the nuclear symmetry energy $E_{sym}(\rho)$. It is well-known that while the maximum mass of neutron stars is determined mostly by the stiffness of the symmetric nuclear EOS $E(\rho, \delta = 0)$, the radii of neutron stars are mostly determined by the slope of the $E_{sym}(\rho)$ [12]. One of the most uncertain part of the EOS of neutron-rich nuclear matter is the density dependence of the $E_{sym}(\rho)$ especially at supra-saturation densities [14]. In the previous studies of the eigen-frequencies of the w-mode using various EOSs [5, 9, 36, 37], the focus was on exploring effects of the stiffness of the EOS and understanding the scaling behavior of the eigen-frequencies. It is still not clear what are the effects of the $E_{sym}(\rho)$. In this study we use EOSs with the same incompressibility K at normal density but different $E_{sym}(\rho)$. Our study is thus complementary to the existing studies [5, 9, 36, 37]. Moreover, the EOS of neutron-rich nuclear matter also plays an important role in heavy-ion collisions especially those induced by neutron-rich radioactive beams in terrestrial laboratories. Significant progress has been made recently in constraining the EOS of neutron-rich nuclear matter using heavy-ion experiments, see, e.g., refs. [14, 22, 23]. In particular, experimental data on collective flow and kaon production in relativistic heavy-ion collisions have put a strong constraint on the EOS of symmetric nuclear matter at densities up to about 5 times the normal nuclear matter density [22]. On the other hand, the $E_{sym}(\rho)$ in the sub-saturation density region has also been constrained recently by analyzing data on isospin diffusion [38, 39, 40, 41] and isoscaling [42, 43] in heavy-ion reactions at intermediate energies. In this work, we use the MDI (Momentum Dependent Interaction) EOSs obtained from Hartree-Fock calculations using a modified Gogny interaction [44]. The parameters of the interactions are adjusted such that the resulting EOSs satisfy all of the above constraints from heavy-ion reactions.

For the modified Gogny MDI interaction, the baryon potential energy density can be expressed as [44]

$$V(\rho,\delta)=\frac{A_u(x)\rho_n\rho_p}{\rho_0}+\frac{A_l(x)}{2\rho_0}(\rho_n^2+\rho_p^2)+\frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^\sigma}(1-x\delta^2)+\frac{1}{\rho_0}\sum_{\tau,\tau'}C_{\tau,\tau'}\int\int d^3pd^3p'\frac{f_\tau(\vec{r},\vec{p})f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}. \tag{24}$$

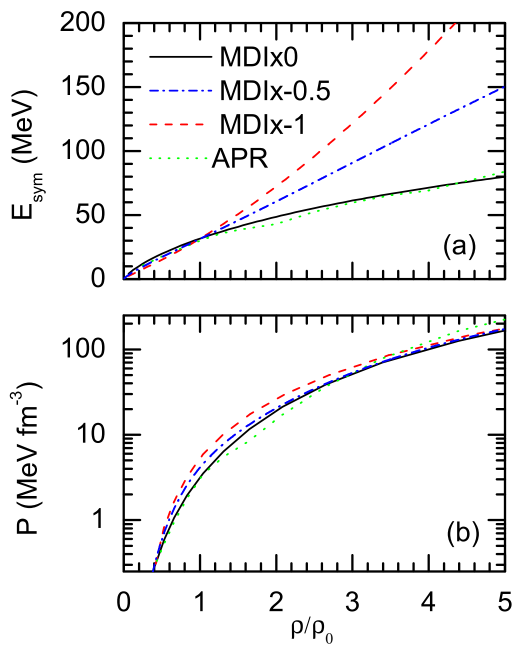

FIG. 1: (Color online) The symmetry energy (a) and pressure (b) versus the baryon number density (baryon number density ρ is in units of saturation nuclear density ρ_0).

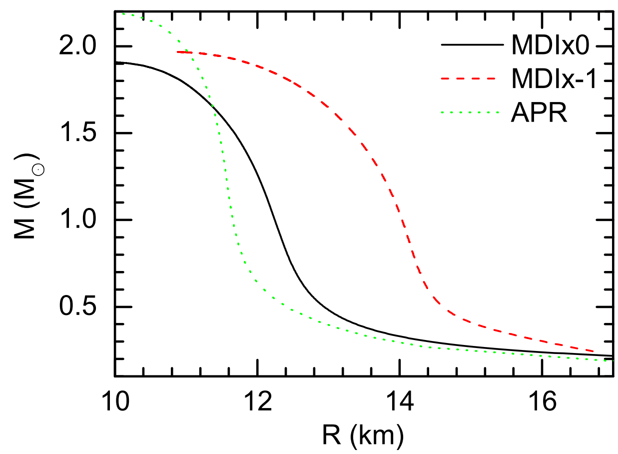

FIG. 2: (Color online) Mass-radius relation for static neutron stars.

In the above equation, the isospin $\tau = 1/2$ $(-1/2)$ is for neutrons (protons). The coefficients $A_u(x)$ and $A_l(x)$ are [40]

$$A_u(x) = -95.98 - x\frac{2B}{\sigma+1}, \quad A_l(x) = -120.57 + x\frac{2B}{\sigma+1}. \tag{25}$$

The values of the parameters are $\sigma = 4/3$, $B = 106.35$ MeV, $C_{\tau,\tau} = -11.70$ MeV, $C_{\tau,-\tau} = -103.40$ MeV and $\Lambda = p_f^0$ which is the Fermi momentum of nuclear matter at ρ_0. The parameter x was introduced to mimic various $E_{sym}(\rho)$ predicted by different microscopic many-body theories. By adjusting the x parameter, the $E_{sym}(\rho)$ is varied without changing any property of symmetric nuclear matter and the symmetry energy at saturation density as the x-dependent $A_u(x)$ and $A_l(x)$ are automatically adjusted accordingly. We note especially that the symmetry energy at normal density $E_{sym}(\rho_0)$ is fixed at 31.6 MeV and the incompressibility K of symmetric nuclear matter at normal density is fixed at 211 MeV consistent with known experimental constraints [14]. We stress that both the $E_{sym}(\rho_0)$ and K are independent of the x parameter. It was shown that [41, 45] only values of x in the range between -1 (MDIx-1) and 0 (MDIx0) are consistent with the isospin-diffusion and isoscaling data as well as the available measurements of the neutron-skin thickness of ^{208}Pb. These data only constrain the $E_{sym}(\rho)$ at sub-saturation densities. At supra-saturation densities, some experimental indications on the trend of the $E_{sym}(\rho)$ has just start to appear[24]. For this exploratory study we assume that the two limiting functions of $E_{sym}(\rho)$ with $x = 0$ and $x = -1$ experimentally constrained only at sub-saturation densities can be extrapolated to supra-saturation densities according to the MDI predictions. Hopefully, future theoretical and experimental studies will allow us to better constrain also the high-density symmetry energy and thus relax the above assumption. Moreover, as we indicated earlier, extrapolating hadronic EOS to high densities without considering the possible hadron-QGP (Quark-Gluon-Plasma) phase transition is always a problem for neutron star models. Given our limited knowledge about the hadron-QGP phase transition and other properties of super-dense matter, for our purpose of studying the imprints of nuclear symmetry energy on the w-mode, we stick to the simplest model of neutron stars made of the *npe* matter.

For comparisons, we also employ the widely used EOS by Akmal, Pandharipande and Ravenhall (APR) [46]. Shown in Fig. 1(a) is the nuclear symmetry energy considered in this work. It is interesting to note that the APR prediction on the $E_{sym}(\rho)$ at sub-saturation den-

TABLE I: Saturation properties of the symmetric nuclear matters EOS. The first column identifies the EOS. The remaining columns presenting the following quantities at nuclear saturation density: saturation baryon density; energy per particle; incompressibility; nucleon effective mass; symmetry energy.

EOS	$\rho_0(fm^{-3})$	$E_s(MeV)$	$K(MeV)$	$m(MeV/c^2)$	$E_{sym}(MeV)$
MDI	0.160	-16.08	211.00	629.08	31.62
APR	0.160	-16.00	266.00	657.25	32.60

sities lies right between the MDIx0 and MDIx-1 predictions. At supra-saturation densities it follows closely the MDI $E_{sym}(\rho)$ with $x = 0$. At nuclear densities below approximately 0.07 fm^{-3}, we supplement the EOSs by those of Refs. [47, 48] which are more suitable for the neutron star crust. The saturation properties of symmetric nuclear matter for the EOSs used here are summarized in Table I. We emphasize that the APR predicts a significantly stiffer incompressibility of 266 MeV compared to the MDI EOS which has an incompressibility of 211 MeV independent of the symmetry energy parameter x.

Shown in Fig. 1(b) is the pressure of the *npe* matter as a function of the baryon density. As one expects, the stiffer symmetry energy with $x = -1$ leads to a higher pressure compared to $x = 0$. As a result, the radii of neutron stars are consistently larger with $x = -1$ as shown in Fig. 2. We notice that the MDIx0 and MDIx-1 EOSs lead to about the same maximum mass for neutron stars. This is because they have the same incompressibility K of 211 MeV for symmetric nuclear matter at normal density. The APR EOS leads to a significantly larger maximum mass because of its stiffer incompressibility but similar radii as the MDIx0 prediction because of the similar symmetry energy density functionals near the saturation density. We notice here that the range of radii between the MDIx0 and MDIx-1 predictions is much smaller than that spanned by the various EOSs considered in ref. [12]. In the latter the EOSs are different not only in their symmetry energy functionals but also the incompressibilities for symmetric nuclear matter.

IV. RESULTS AND DISCUSSION

In this section we present our numerical results for the first axial w-mode (w_I-mode), the w_{II}-mode, the second w-mode (w_{I2}-mode), and some of the third axial w-modes (w_{I3}-modes). The calculation is performed applying the continued fraction method together with the EOSs we discussed in the previous Section.

We first examine effects of the symmetry energy on the frequencies and damping times of the w-mode. Shown in Figs. 3 and 4 are the frequency and damping time of the w_I-mode (a) and w_{II}-mode (b) respectively, as functions of the neutron star mass. These figures establish the relationship between the expected frequencies of the

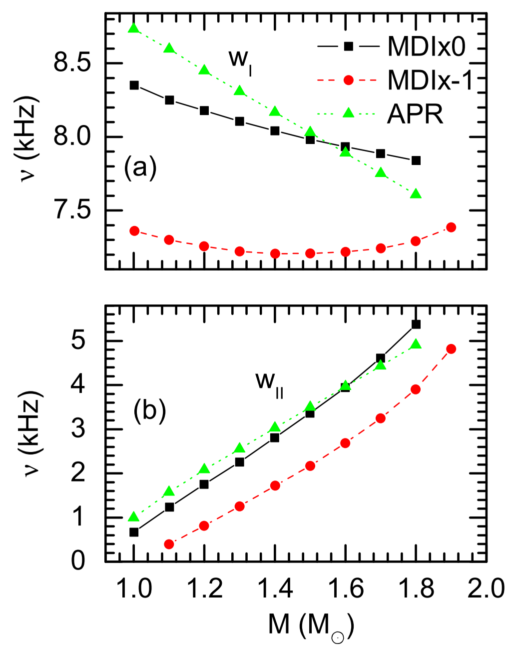

FIG. 3: (Color online) Frequency of w_I-mode (a) and w_{II}-mode (b) as a function of the neutron star mass M.

axial w-modes, for a given EOS, and the stellar mass. It is interesting to notice, in Fig. 3, that there is a clear difference between the frequencies calculated with the MDIx0 EOS and those with the MDIx-1 EOS. Since the major difference between these two cases is the density dependence of the nuclear symmetry energy, it is obvious that the symmetry energy has a clear imprint on the frequencies. Therefore, the symmetry energy constrained by nuclear reactions in terrestrial laboratories can help determine the expected frequency of the axial w-mode and its damping time.

In Figures 5, 7 and 9 we show the real (a) and imaginary (b) parts of the eigen-frequency of w_I-, w_{I2}- and w_{II}-modes scaled by the mass M as a function of the neutron star compactness M/R, respectively. These results suggest that the scaled eigen-frequency exhibits a universal behavior independent of the EOS used as a function of the compactness parameter. Similar universal behaviors were first found for the polar w-mode by Andersson et al.[5] and later for the axial w-mode by Benhar et al. [49] and Tsui et al. [36, 37]. As it was pointed out earlier in the above references, provided that the masses and radii of neutron stars are known, the universal scaling behaviors allows an accurate determination of the w-mode frequency and damping time of gravitational waves. This is very important for guiding the gravitational wave search. On the other hand, if both the frequency and damping time for a given neutron star are known this could provide information on the neutron star mass and radius. In Figs. 5 and 7 we also display the curves (Fit) best fitting

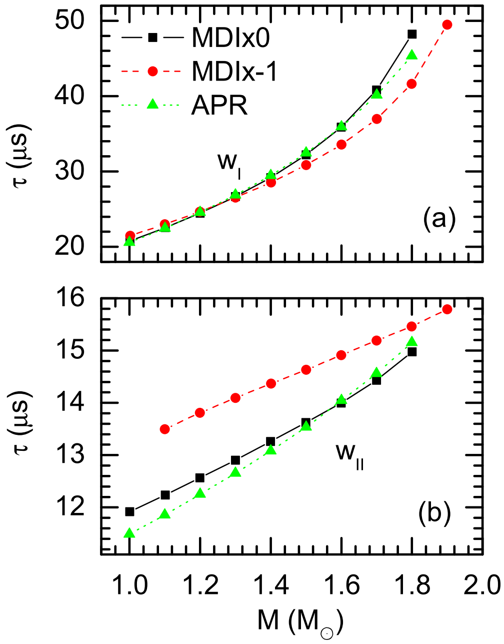

FIG. 4: (Color online) Damping time of w_I-mode (a) and w_{II}-mode (b) as a function of the neutron star mass M.

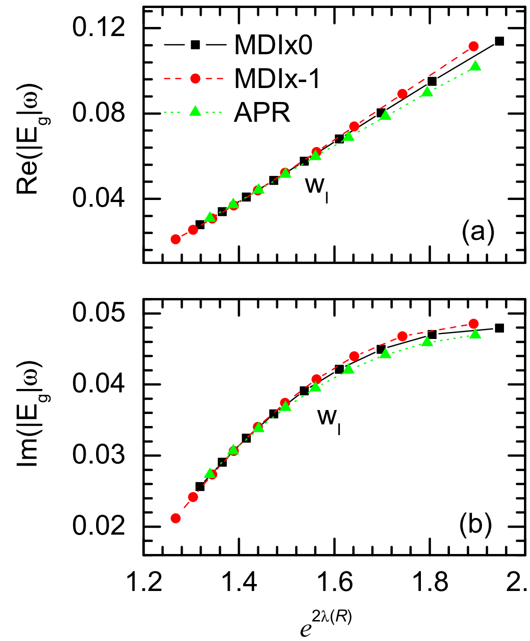

FIG. 6: (Color online) Eigen-frequency, ω, of the w_I-mode scaled by the gravitational energy $|E_g|$ versus the metric function $e^{2\lambda}$ at the stellar surface.

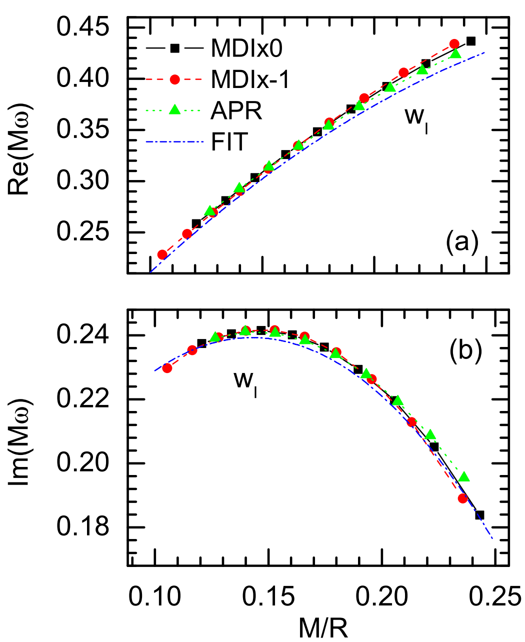

FIG. 5: (Color online) Eigen-frequency, ω, of the w_I-mode scaled by the stellar mass M as a function of compactness M/R. The fit is performed by employing the parameters of Tsui et al. [36].

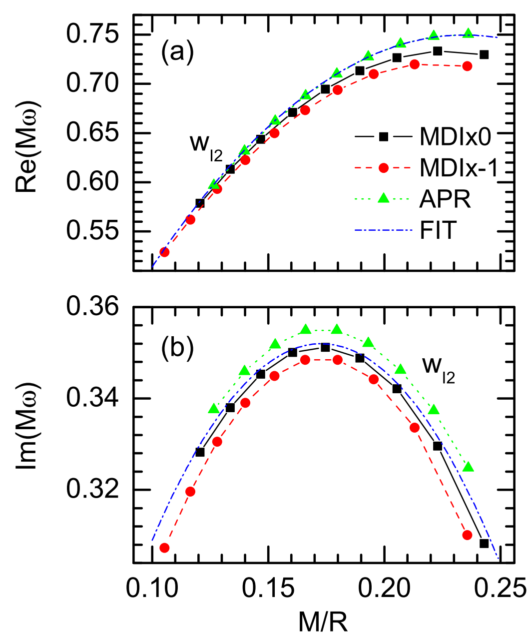

FIG. 7: (Color online) Eigen-frequency, ω, of the w_{I2}-mode scaled by the stellar mass M as a function of compactness M/R. The fit is performed by employing the parameters of Tsui et al. [36].

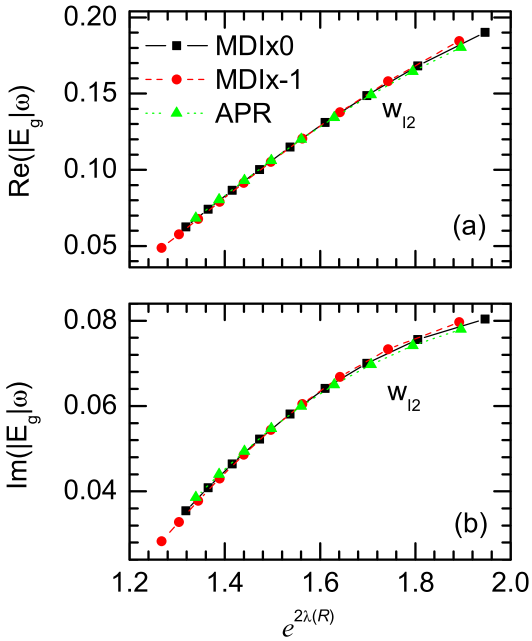

FIG. 8: (Color online) Eigen-frequency, ω, of the w_{I2}-mode scaled by the gravitational energy $|E_g|$ versus the metric function $e^{2\lambda}$ at the stellar surface.

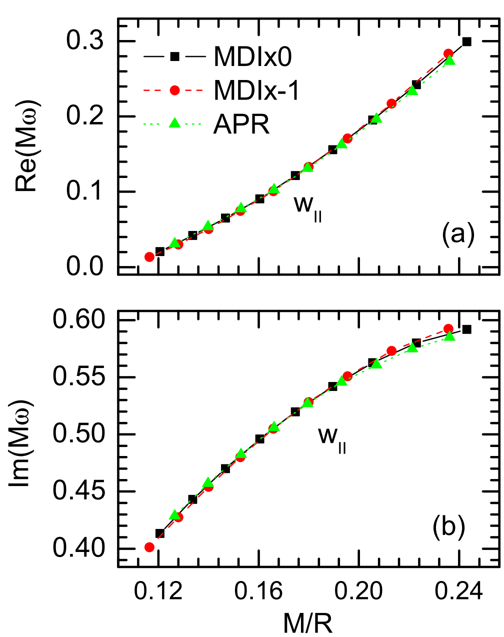

FIG. 9: (Color online) Eigen-frequency, ω, of the w_{II}-mode scaled by the stellar mass M as a function of compactness M/R.

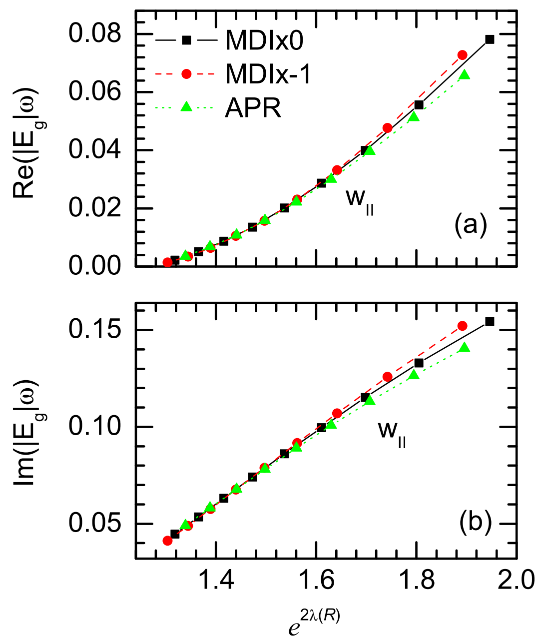

FIG. 10: (Color online) Eigen-frequency, ω, of the w_{II}-mode scaled by the gravitational energy $|E_g|$ versus the metric function $e^{2\lambda}$ at the stellar surface.

the results obtained by Tsui et al. using eight different EOSs [36]. It is seen that our numerical results are in good agreement with theirs.

Realizing that the axial w-mode is a space-time mode [9], it is interesting to study whether the w-mode eigen-frequencies may be scaled by the total gravitational energy and how the scaled frequencies behave as functions of the metric function $e^{2\lambda(R)} = 1/(1-2M/R)$ (at the stellar surface). This approach relates again the eigen-frequency to the compactness parameter M/R of neutron stars, but additionally it has the advantage of taking into account more completely the space-time properties. The gravitational energy is calculated from [50]

$$E_g = \int_0^R 4\pi r^2 \{1 - [1 - \frac{2m(r)}{r}]^{-1/2}\} \rho dr. \tag{26}$$

Shown in Figs. 6, 8 and 10 are the real (a) and imaginary (b) parts of the eigen-frequencies of the w_I-, w_{I2}- and w_{II}-modes scaled by the absolute value of the gravitational energy $|E_g|$ as functions of the metric function $e^{2\lambda(R)}$ at the surface. Compared to the corresponding frequencies scaled by the neutron star mass as functions of M/R, while the universal behavior of the w_I- and w_{II}-mode frequencies remain about the same, the w_{I2}-mode frequencies scaled by the total gravitational energy as functions of $e^{2\lambda(R)}$ become much more universal.

Besides the above two kinds of universal scalings, it is possible to obtain alternative scalings. One particularly

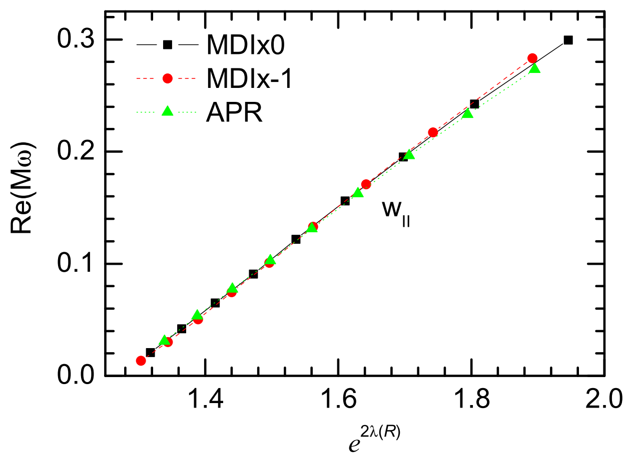

FIG. 11: (Color online) Real part of the scaled frequency $M\omega$ of w_{II}-mode versus the metric function $e^{2\lambda}$ at the stellar surface.

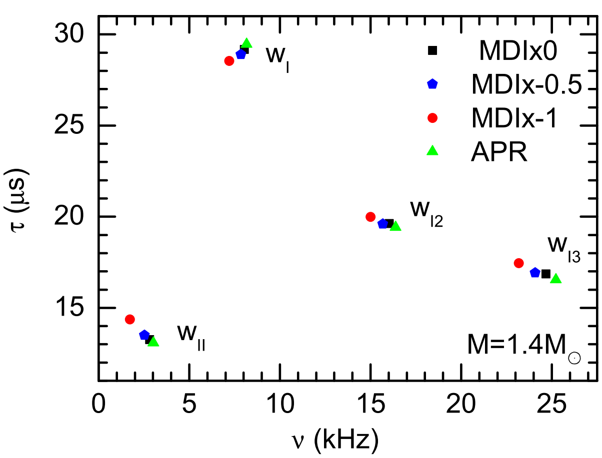

FIG. 12: (Color online) Frequency and damping time of the first, second and third axial w-mode (w_I, w_{I2} and w_{I3}) and the axial w_{II}-mode for neutron star models of mass $M = 1.4M_\odot$.

useful scaling we observed is that the real frequency ω of the w_{II}-mode scaled by the mass M varies linearly and independent of the EOS as a function of the metric function $e^{2\lambda(R)}$, see Fig. 11. From the lower panel of Fig. 3, one can see that there exists a minimum mass or compactness, below which the w_{II}-mode frequency will vanish, namely, there is no w_{II}-mode below the limit. Unfortunately, it is not easy to obtain this limit directly based on Fig. 3. However, the linear characteristic shown in Fig. 11 allows one to easily and accurately determine the minimum compactness for the existence of the w_{II}-mode to be $M/R \approx 0.1078$.

To be more quantitative and make it easier to compare with future studies, we summarize in Table II several neutron star properties, the corresponding eigen-frequencies and damping times for the EOSs considered in this work.

Finally, in Fig. 12 we show both the frequencies and the damping times of the w_I-, w_{II}-, w_{I2}-, and w_{I3}-modes for canonical neutron stars of mass $M = 1.4M_\odot$. In order to evaluate their dependence on the symmetry energy, we used all four symmetry energy functionals shown in Fig.1, namely $x = 0, -0.5, -1$ and the APR, that are within the constraints set by the heavy-ion reaction data. It is seen that the various modes have different dependences on the symmetry energy. While the damping time of the w_I mode increases with the increasing frequency as the symmetry energy becomes softer (from $x = -1$ to $x = 0$ as shown in Fig. 1), the opposite behaviors are observed for the w_{12}, w_{13} and w_{II} modes. Thus, simultaneous studies of multiple w-modes will be useful in understanding the imprints of symmetry energy on the w-modes of gravitational waves. It is also seen that the w_{II}-mode has the smallest frequency and damping time - the frequency of w_{II}-mode for a neutron star of mass $M = 1.4M_\odot$ with the $x = -1$ EOS is about 1.72 kHz.

V. SUMMARY

In summary, we have examined the eigen-frequencies of the first few axial w-modes of oscillating neutron stars by using an EOS with the symmetry energy partially constrained by the recent terrestrial nuclear laboratory data. Our studies indicate that the density dependence of the nuclear symmetry energy $E_{sym}(\rho)$ has a clear imprint on both the frequency and the damping time of the axial w-modes. We confirmed the previously found universal behavior of the mass-scaled eigen-frequencies. Moreover, we explored several alternative universal scalings of the eigen-frequencies. The latter scaled with the absolute value of the gravitational energy $|E_g|$ are more universal functions independent of the symmetry energy, especially for the w_{I2}-mode. Furthermore, it is found that for the w_{II}-mode to exist neutron stars have to have a minimum compactness of $M/R \approx 0.1078$ independent of the EOS used.

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TABLE II: Neutron star properties (central density, radius, mass, compactness and gravitational energy), and eigen-frequencies and damping times of the w_I- and w_{II}- modes for the EOSs considered in this paper.

EOS	$\rho_c(10^{18}kg \cdot m^{-3})$	$R(km)$	$M(M_\odot)$	M/R	$\lvert E_g\rvert(M_\odot)$	$\nu_{w_I}(kHz)$	$\tau_{w_I}(\mu s)$	$\nu_{w_{II}}(kHz)$	$\tau_{w_{II}}(\mu s)$
MDIx0	0.7265	12.25	1.00	0.121	0.108	8.35	20.8	0.67	11.9
	0.7970	12.16	1.10	0.134	0.133	8.25	22.6	1.23	12.2
	0.8728	12.07	1.20	0.147	0.161	8.18	24.5	1.75	12.6
	0.9586	11.96	1.30	0.160	0.194	8.11	26.7	2.26	12.9
	1.0595	11.83	1.40	0.175	0.232	8.04	29.2	2.81	13.3
	1.1780	11.68	1.50	0.190	0.276	7.98	32.2	3.36	13.6
	1.3196	11.49	1.60	0.205	0.327	7.93	35.9	3.95	14.0
	1.4969	11.25	1.70	0.223	0.390	7.89	40.8	4.61	14.4
	1.7760	10.93	1.80	0.243	0.469	7.84	48.2	5.36	15.0
MDIx-1	0.5535	13.94	1.10	0.116	0.113	7.30	23.0	0.39	13.5
	0.6140	13.83	1.20	0.128	0.137	7.26	24.7	0.81	13.8
	0.6835	13.70	1.30	0.140	0.165	7.22	26.5	1.25	14.1
	0.7630	13.54	1.40	0.153	0.197	7.21	28.5	1.72	14.4
	0.8570	13.35	1.50	0.166	0.234	7.21	30.8	2.17	14.6
	0.9700	13.12	1.60	0.180	0.278	7.22	33.6	2.69	14.9
	1.1181	12.84	1.70	0.195	0.330	7.24	37.0	3.25	15.2
	1.3220	12.47	1.80	0.213	0.395	7.29	41.6	3.90	15.5
	1.6760	11.90	1.90	0.236	0.488	7.39	49.5	4.82	15.8
APR	0.7880	11.66	1.00	0.127	0.114	8.73	20.6	0.99	11.5
	0.8383	11.62	1.10	0.140	0.140	8.60	22.5	1.57	11.9
	0.8895	11.58	1.20	0.153	0.169	8.45	24.6	2.08	12.2
	0.9422	11.55	1.30	0.166	0.201	8.31	26.9	2.55	12.7
	1.0019	11.51	1.40	0.180	0.237	8.17	29.5	3.03	13.1
	1.0665	11.46	1.50	0.193	0.277	8.03	32.4	3.50	13.5
	1.1370	11.41	1.60	0.207	0.322	7.89	35.9	3.97	14.0
	1.2155	11.32	1.70	0.221	0.374	7.75	40.1	4.43	14.6
	1.3140	11.25	1.80	0.236	0.432	7.61	45.4	4.91	15.2

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